Exercise 65

(a) Use a graph to estimate the absolute maximum and minimum values of the function to two decimal places.

(b) Use calculus to find the exact maximum and minimum values.

$$f(x) = x^5 - x^3 + 2, \; -1 \leq x \leq 1$$

Solution

Take the derivative of the function.

$$f'(x) = \frac{d}{dx}(x^5 - x^3 + 2)$$

$$= 5x^4 - 3x^2$$

Set $f'(x) = 0$ and solve for x.

$$5x^4 - 3x^2 = 0$$

$$x^2(5x^2 - 3) = 0$$

$$x^2 = 0 \quad \text{or} \quad 5x^2 - 3 = 0$$

$$x = 0 \quad \text{or} \quad x^2 = \frac{3}{5}$$

$$x = 0 \quad \text{or} \quad x = -\sqrt{\frac{3}{5}} \approx -0.774597 \quad \text{or} \quad x = \sqrt{\frac{3}{5}} \approx 0.774597$$

$x = 0$ and $x = \pm\sqrt{3/5}$ are within $-1 \leq x \leq 1$, so evaluate f at these values. The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $-1 \leq x \leq 1$.

$$f(0) = 0^5 - 0^3 + 2 = 2$$

$$f\left(-\sqrt{\frac{3}{5}}\right) = \left(-\sqrt{\frac{3}{5}}\right)^5 - \left(-\sqrt{\frac{3}{5}}\right)^3 + 2 = 2 + \frac{6}{25}\sqrt{\frac{3}{5}} \approx 2.1859 \qquad \text{(absolute maximum)}$$

$$f\left(\sqrt{\frac{3}{5}}\right) = \left(\sqrt{\frac{3}{5}}\right)^5 - \left(\sqrt{\frac{3}{5}}\right)^3 + 2 = 2 - \frac{6}{25}\sqrt{\frac{3}{5}} \approx 1.8141 \qquad \text{(absolute minimum)}$$

The graph below illustrates these results.